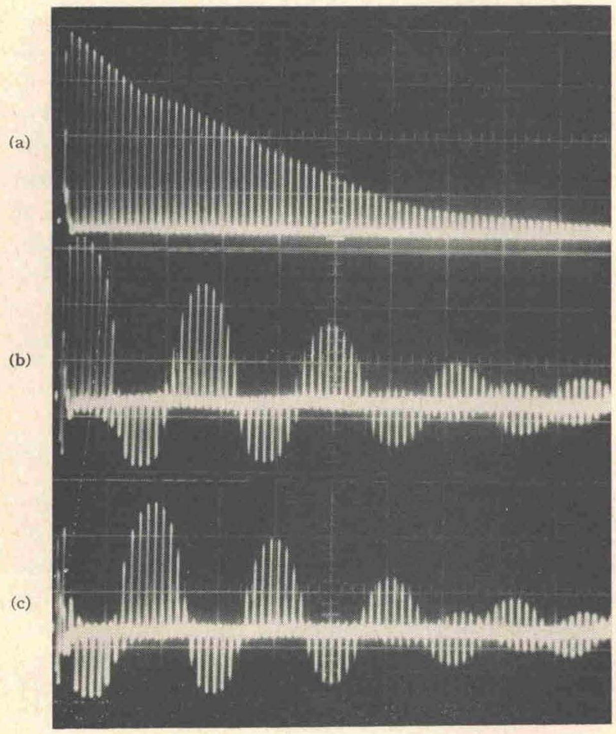

FIG. 5. Oscillogram of coherent pulse/cw signals.

now mixed with the pulse echo signal, interference between the two signals will occur. Constructive interference will result when the phase difference between the two signals is $r\pi$ radians, where r is an even integer. For all other phase differences, destructive interference will occur, maximum interference resulting when r is an odd integer. By adjusting the level of cw to approximately that of the early echoes in the pulse echo system and varying the frequency of the VFO, frequencies corresponding to $\nu_{(n-m)}, \ldots \nu_n, \ldots, \nu_{(n+p)}$ of the cw technique can be found. These frequencies correspond to phase differences of $r\pi$ rad, where r is an even integer, that is when the pulse echo and cw signals are in phase. When the cw and pulse signals are out of phase, a sinusoidal modulation signal appears on the pulse echo envelope. The VFO frequency at which zero modulation occurs can rapidly be found with great accuracy by adjusting the VFO frequency and observing the pulse echo pattern. Figure 5 shows three oscillograms obtained when the VFO is tuned (a) to a frequency ν_n, for zero modulation, i.e., when the phase difference between pulse and cw signals is exactly $r\pi$ rad and r is an even integer; (b) to a slightly greater frequency $(\nu_n+\delta\nu)$ so that the phase difference is approximately 0.03 rad greater than $r\pi$; (c) to a slightly lower frequency $(\nu_n-\delta\nu)$, so that the phase difference is approximately 0.03 rad less than $r\pi$. For these oscillograms, $\nu_n=10.2202$ Mc, $\Delta\nu_{av}=0.1433$ Mc $\delta\nu\simeq 5$ kc. It should be noted that the nonsinusoidal modulation of the echo envelope of Fig. 5(a) is not due to the mixing of cw and pulse signals, but rather to the geometry of the sample and possibly to diffraction effects.[7] The greatest sensitivity in determining the frequencies for which the phase difference between pulse and cw signals is zero is achieved when the amplitude of the cw signal is made approximately equal to the amplitude of the early echoes in the pulse echo train as shown in Fig. 5.

The velocity of sound v can then be calculated using either Eq. (1) or Eqs. (2) and (3) of the cw method. Equation (1) can be simplified to

$$v=2\ell_S\Delta\nu_{av}, \quad (4)$$

if the accuracy of velocity measurement is required to within a few parts in 10^2. This method can only be adopted when $m_T/m_S<10^{-2}$.

5. INSTRUMENTATION

The equipment used in the coherent pulse/cw technique is shown in block diagram form in Fig. 4. Pulses of rf are generated by gating and amplifying the output of a VFO. The amplified rf pulses are coupled, via a matching network, to the quartz transducer, which is bonded to the sample. The matching network also serves to match the receiver to the transducer and to the cw rf source. The rf attenuator can be varied to adjust the level of cw signal being mixed with the pulse echo signals. The resultant signal is amplified by a wide band rf amplifier and subsequently detected before being displayed on a cathode ray tube. An electronic counter is used to measure the VFO frequency. A permanent record of the frequencies $\nu_{(n-m)}, \ldots \nu_n, \ldots \nu_{(n+p)}$ can be obtained by using a digital printer coupled to the electronic counter.

Most of the units indicated in Fig. 4 are commercially available. Those used for the measurements described here are the following: General Radio unit oscillator type 1211B, Hewlett-Packard step attenuators types 355A and 355B, Beckman electronic counter type 7170, Beckman digital printer type 1453, Measurements Corporation pulse generator type 79B, Tektronix oscilloscope type 585. The

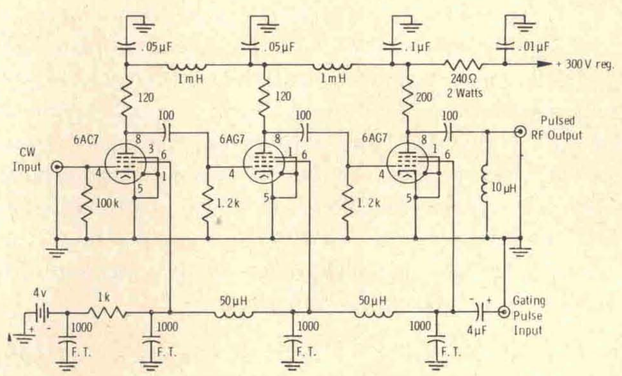

FIG. 6. Gated rf power amplifier.

[7] A. Seki, A. Granato, and R. Truell, J. Acoust. Soc. Am. 28, 230 (1956).

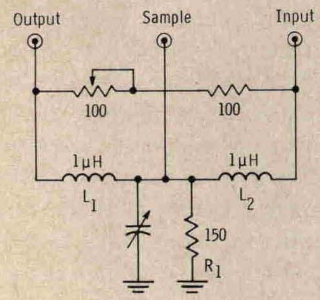

FIG. 7. Matching network.

wide band rf amplifier and detector was a modified surplus radar receiver. This unit can readily be replaced by a tuned commercial amplifier with a bandwidth of several megacycles and a gain of approximately 80 dB. The circuit of the gated rf power amplifier is shown in Fig. 6. The screen grid of each stage is held at a negative potential of 4 V. This bias is sufficient to hold each stage at cutoff. When a positive-going gating pulse of approximately 150 V is applied to the "gating pulse input" connector, all stages are simultaneously driven up to normal operating conditions, and the rf input signal is amplified for this short duration. Rejection of the cw signal between gating pulses is approximately 90 dB. This simple circuit provides adequate rf power when the maximum output from the VFO is coupled to the input. The circuit diagram of the matching network is given in Fig. 7. The network shown is for 20 Mc. L_1, L_2, and R_1 are switched to appropriate values when used at other frequencies. Connection to the cw source is made via a BNC T-connector to the "output" terminal.

6. RELATIONS BETWEEN ACOUSTIC VELOCITIES, ELASTIC CONSTANTS, AND DEBYE TEMPERATURE FOR CUBIC CRYSTALS

A strong analogy exists between the propagation of elastic and light waves in crystals, although the three surfaces—velocity, inverse, and wave—are considerably more complicated in the mechanical case. An expression of the condition for the existence of plane elastic waves in an anisotropic medium leads to a cubic equation in ρv^2, the elastic constants, and the direction of the wave normal, which yields three real positive roots for any value of (ℓ,m,n), the direction cosines of the wave normal. Associated with each velocity is a uniquely defined displacement vector, one quasilongitudinal and two quasitransverse. These three vectors form an orthogonal triad. The plot of $v(\ell,m,n)$ gives the velocity surface for any medium; for a cubic crystal the velocity equation is

$$H^3-aH^2+c(a+b)AH-c^2(a+2b)B=0, \qquad (5)$$

where $H=\rho v^2-c_{44}$; $a=c_{11}-c_{44}$; $b=c_{12}+c_{44}$; $c=c_{11}-c_{12}-2c_{44}$; $A=m^2n^2+n^2\ell^2+\ell^2m^2$; $B=\ell^2m^2n^2$; ρ is the density; v is the velocity.

In general, the normal to the wave surface does not coincide with the radius vector, or wave normal, and the propagation of energy takes place along three extraordinary rays; only when the wave normal and the normal to the wave surface coincide does the energy travel along the wave normal. A crystal of cubic symmetry possesses three such directions, viz., the [100], [110], and [111] types of direction. Hence, for propagation in the [100] direction, each ray is an ordinary ray, and as $\ell=1$, $m=n=0$, Eq. (5) yields the velocities

$$v_L=(c_{11}/\rho)^{\frac{1}{2}}, \qquad (6)$$

where v_L is the velocity of compressional waves and

$$v_{T_1}=v_{T_2}=(c_{14}/\rho)^{\frac{1}{2}}, \qquad (7)$$

where v_{T_1} and v_{T_2} are the velocities of the two shear waves.

For propagation in the [110] direction, each ray is an ordinary ray, as the wave normal and wave surface normal coincide. For this direction $\ell=m=1/\sqrt{2}$ and $n=0$ and Eq. (5) yields three unique velocities,

$$v_L=[(c_{11}+c_{12}+2c_{44})/2\rho]^{\frac{1}{2}}, \qquad (8)$$

$$v_{T_1}=(c_{44}/\rho)^{\frac{1}{2}}, \qquad (9)$$

$$v_{T_2}=[(c_{11}-c_{12})/2\rho]^{\frac{1}{2}}. \qquad (10)$$

For propagation in the [111] direction, $\ell=m=n=1/\sqrt{3}$ yielding the following velocities from Eq. (5),

$$v_L=[(c_{11}+2c_{12}+4c_{44})/3\rho]^{\frac{1}{2}}, \qquad (11)$$

$$v_{T_1}=v_{T_2}=[(c_{11}-c_{12}+c_{44})/3\rho]^{\frac{1}{2}}. \qquad (12)$$

Along this direction, the wave normal coincides with the normal to the compressional velocity surface, and hence energy propagates along the wave normal. The velocity of the shear waves is degenerate and consequently the displacement vectors are not uniquely defined, but may be in any direction contained by the plane of the wave. Also the normal to the velocity surface is not uniquely defined but may take up an infinity of positions, associated with different displacement vectors. This results in a cone of extraordinary rays giving rise to internal conical refraction.[8] The semi-angle of this cone is given by

$$\tan\Delta_c=c/(c+3c_{44})\sqrt{2}, \qquad (13)$$

where c is as defined for Eq. (5) and is the normal measure of the degree of elastic anisotropy possessed by a cubic crystal.

The Debye temperature θ can be evaluated from the equation[9]

$$\theta=(h/k)[3qN\rho/4\pi M]^{\frac{1}{3}}v_m, \qquad (14)$$

where h is Planck's constant, k is Boltzmann's constant,

[8] J. de Klerk and M. J. P. Musgrave, Proc. Phys. Soc. (London) **68B**, 81 (1955).

[9] O. L. Anderson, J. Phys. Chem. Solids **24**, 909 (1963).